

ICMI Study 23



澳門大學
UNIVERSIDADE DE MACAU
UNIVERSITY OF MACAU

Primary Mathematics Study on Whole Numbers

June 3 - 7, 2015 in Macau / China

**It is time to reveal
what students with MLD know,
rather than what they do not know**

Marja van den Heuvel-Panhuizen



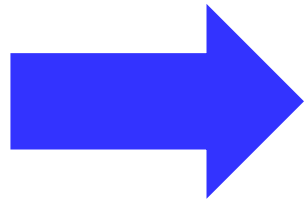
Freudenthal Institute
Universiteit Utrecht

Faculty of Science

Faculty of Social and Behavioural Sciences

Knowing what students know

is *the* starting point of good instruction



Looking for what students know

- implies a change in assessment:
Assessing (MLD) students' mathematical potential
- implies a change in teaching:
Building on what (MLD) students already know

Examples



IMPULSE project

1. Offering students problems in which they can show their competence
 - 1a. Within curriculum: Subtraction problems
 - 1b. Beyond curriculum: Combinatorial problems



FaSMEd project

2. Offering students optional auxiliary tools:
Percentage problems



***“Going across the grain” study* by Watson (2002)**

3. Advanced mathematical thinking by
low attaining students

Example 1a

***Offering students problems
by which they can show their
competence***

***Subtraction problems that
elicit strategies***

Example 1a

Study with Special Education students

56 students from 14 classes in SE school

8–12 years old

Mathematics level Grade 2

Students who are weak in mathematics should be taught just **one procedure**: Subtraction should be solved by Direct Subtraction and not by Indirect Addition

Example 1a

62 euro



29 euro discount

1	2	3	4	5	6	7	8	9	0
---	---	---	---	---	---	---	---	---	---

answer:



Direct Subtraction
DS strategy

**Taking Away
Context**

next 

space for 51 cards



49 are already included

1	2	3	4	5	6	7	8	9	0
---	---	---	---	---	---	---	---	---	---

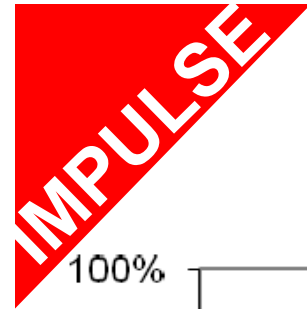
answer:



Indirect addition
IA strategy

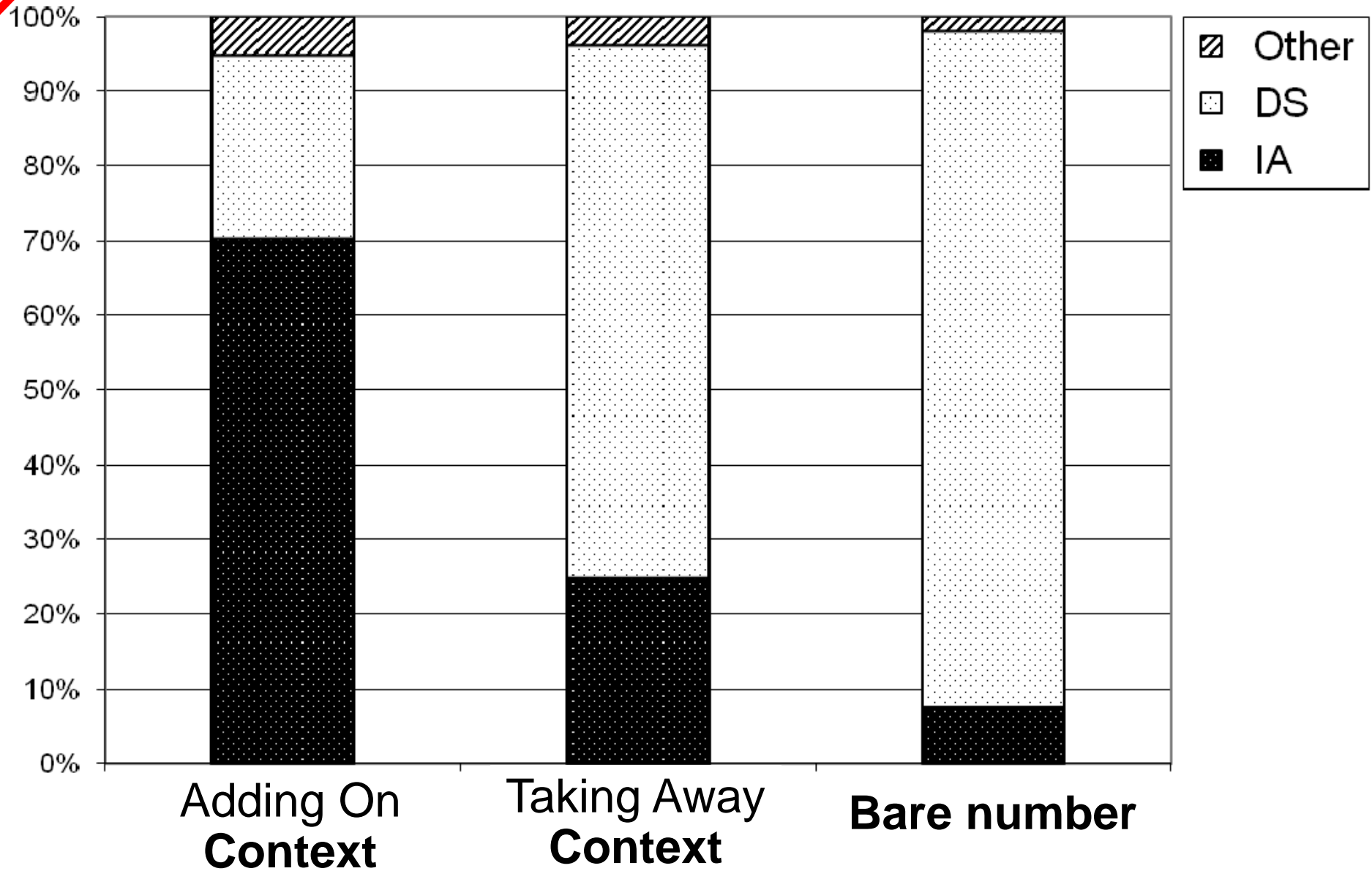
**Adding On
Context**

next 

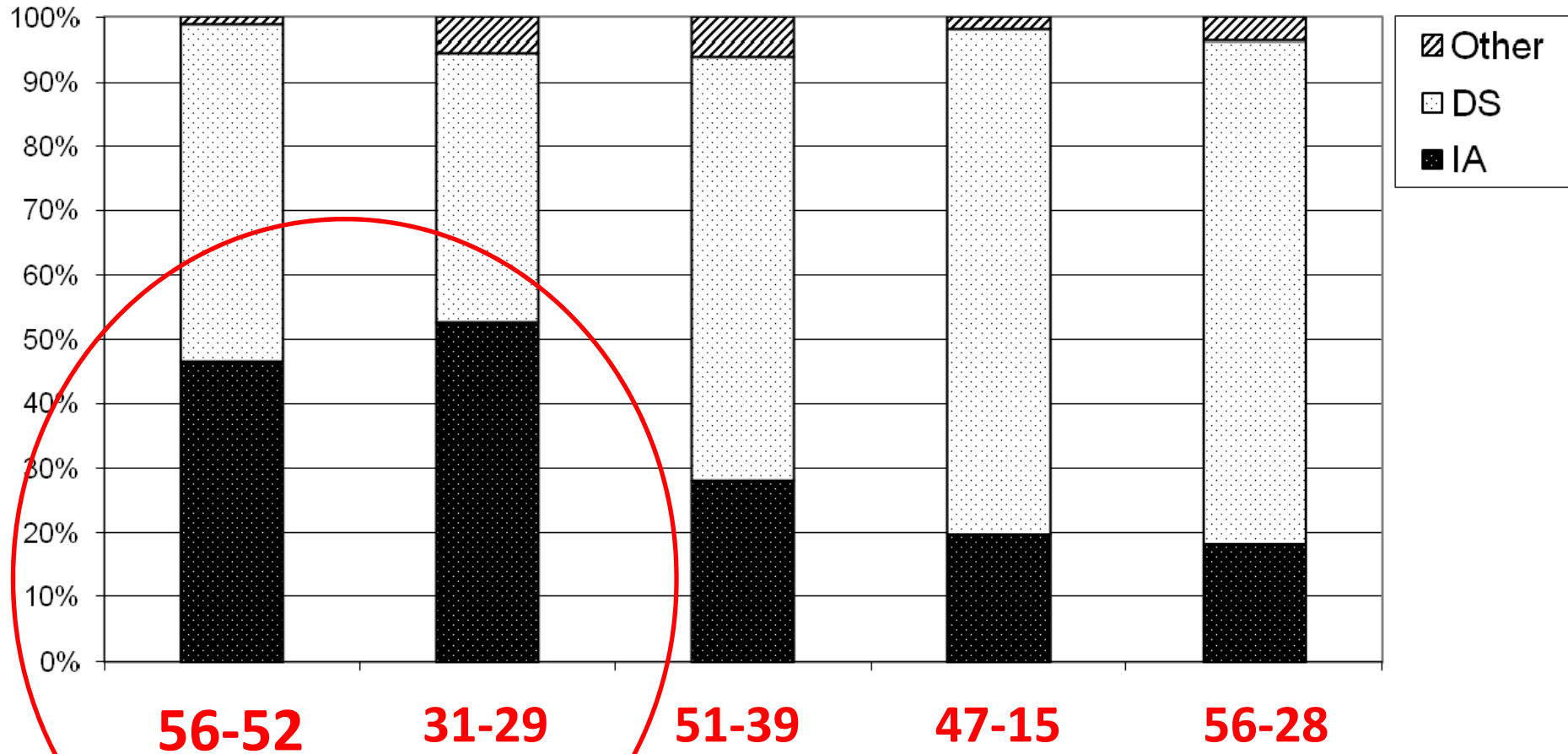


Example 1a

IA use and problem format



IA use and numbers involved



Conclusions

- SE students can make spontaneous use of IA
 - DS 63%
 - IA 34%
 - Average IA use per student 4.6 (max 0, max 8)

- SE students are rather flexible in applying IA

- SE students are quite successful when applying IA
 - DS 51% correct
 - IA 68% correct

Example 1b

***Offering students problems
in which they can show their
competence:***

Combinatorial problems

Example 1b

Research question:

Can special education students solve combinatorial problems?

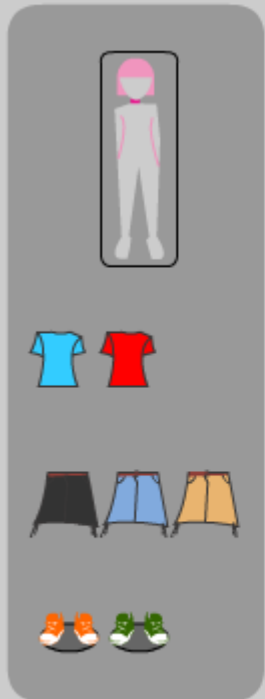
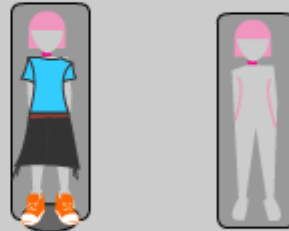
Participants:

84 students (age $M = 11.1$) from 5 SE schools
76 students (age $M = 9.4$) from 5 RE schools
Mathematics levels Grade 2-5

Instrument:

6 combinatorial problems in ICT environment

Example 1b

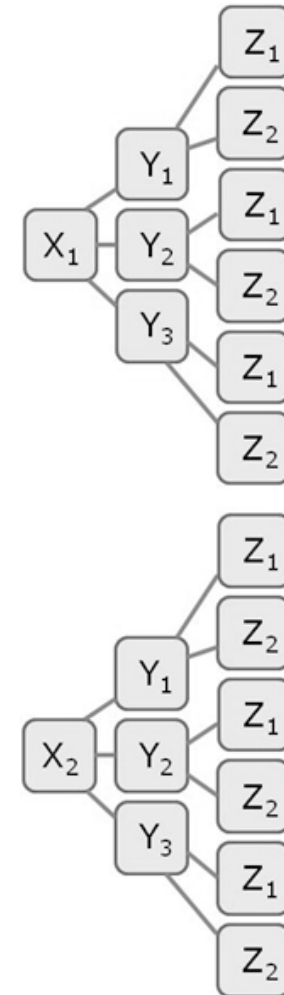
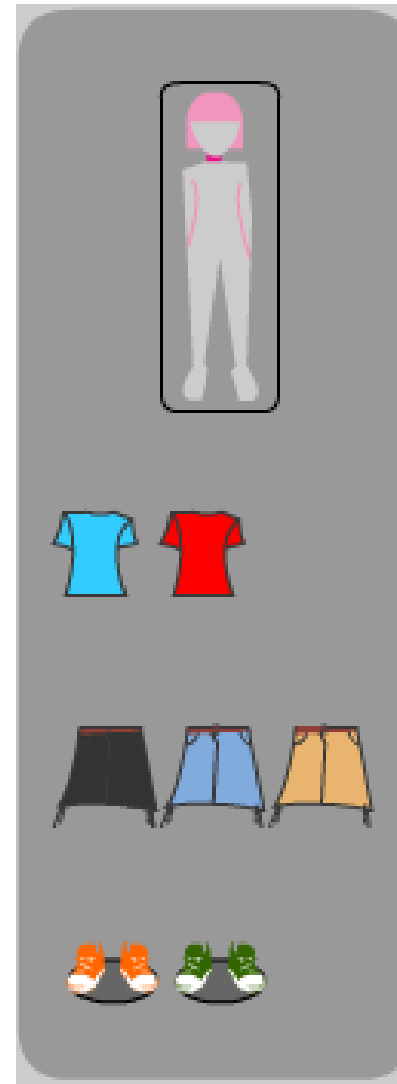


Correctly solved problems	
SE students	RE students
56%	57%

Example 1b

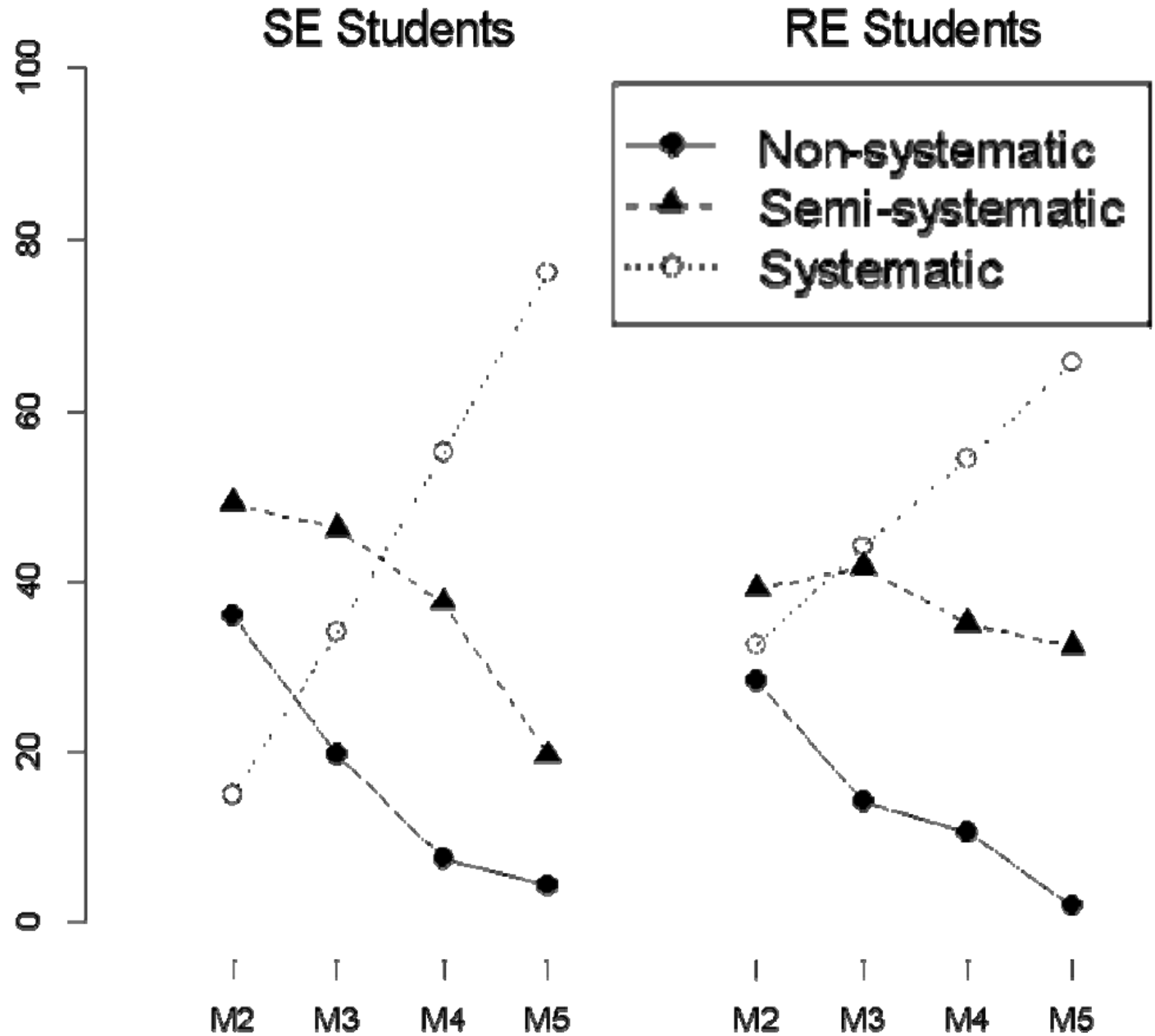
Strategy use:

- systematic
- semi-systematic
- non-systematic



Example 1b

Frequency (%)
of strategy use



Example 2

***Offering students optional
auxiliary tools:***

Percentage problems

Example 2



FaSMEd



Digital Assessment Environment

- Web-based
- Monitoring function
- Problems based on key competencies
- **Auxiliary tools**

Six problems on percentage

Grade 6 teacher:

“Duncan belongs to the low-level stream in my class and now he did three of the six problems correctly!”

Example 2






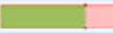


Problem 1

When a battery is full, it will work 120 hours.

It is still charged for 40%.

For how many hours will this battery still work?

Answer: hours

		scrap paper empty
		scrap paper with grid
		bar
		table

The tool icon is clicked

Example 2

Problem 1

When a battery is full, it will work 120 hours.

It is still charged for 40%.

For how many hours will this battery still work?

Answer: hours

The screenshot shows a software interface with a light blue background. At the top, there are three horizontal bars representing different objects: 'scrap paper empty' (with a notepad icon), 'scrap paper with grid' (with a grid icon), and 'bar' (with a green and red bar icon). Below these is a large white area containing a diagram of a battery. The battery is represented by a green rectangle with a red outline. To the right of the battery, there is a vertical line with a red dot at the top labeled '100%' and a black dot at the bottom labeled 'Total'. A blue speech bubble points to the 'Total' label, containing the text 'The purple bullet is moved up'. At the bottom of the interface, there is a horizontal bar labeled 'table' with a table icon.

The purple bullet is moved up

Example 2

Problem 1

When a battery is full, it will work 120 hours.

It is still charged for 40%.

For how many hours will this battery still work?

Answer: hours

scrap paper empty

scrap paper with grid

bar

100%

120

Red bullet is moved to the left

table

Example 2

Problem 1

When a battery is full, it will work 120 hours.
 It is still charged for 40%.
 For how many hours will this battery still work?
 Answer: hours

The answer is filled in

	scrap paper empty
	scrap paper with grid
	bar

40% 100%

48 120

table

Example 2

Problem 1

When a battery is full, it will work 120 hours.

It is still charged for 40%.

For how many hours will this battery still work?

Answer: hours

Diagram illustrating the problem setup using a bar model:

- scrap paper empty**: Represented by a blank notepad icon.
- scrap paper with grid**: Represented by a grid icon.
- bar**: Represented by a horizontal bar with a green segment on the left and a red segment on the right.

The bar model shows a total length of 120 units. The green segment represents 40% of the total, which is 48 units. The red segment represents the remaining 72 units. A vertical slider on the right indicates the percentage of the total length, currently set at 40%.

Segment	Percentage	Hours
Green (Charged)	40%	48
Red (Remaining)	60%	72
Total	100%	120

table: Represented by a table icon.

Example 2

Problem 2

A cell phone costs 70 euro.
You get a discount of 20%.
What do you have to pay?

Answer 66 euro

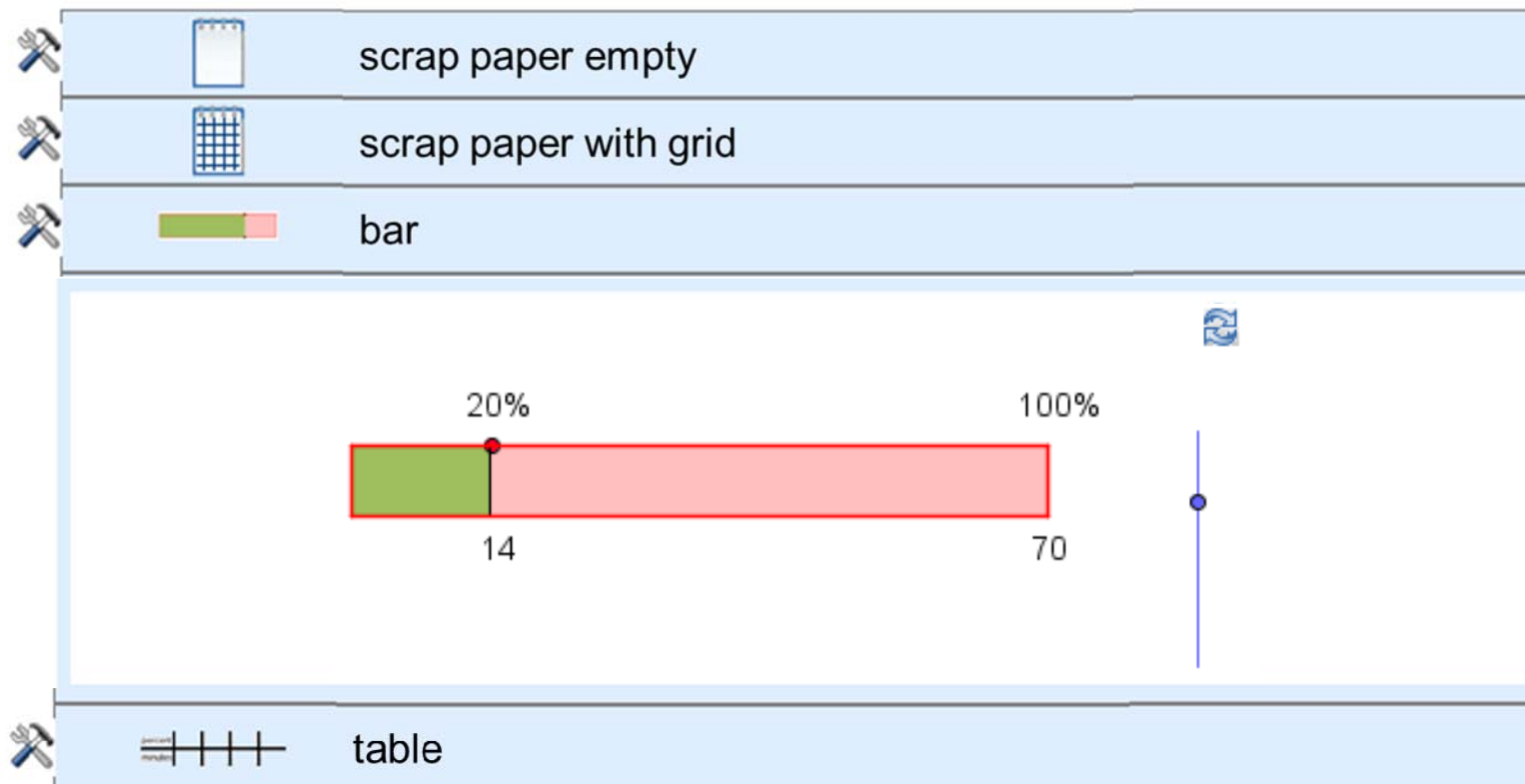
Wrong answer,
but what do the
auxiliary tools
tell the teacher ?

Example 2

Problem 2

A cell phone costs 70 euro.
You get a discount of 20%.
What do you have to pay?

Answer **66** euro



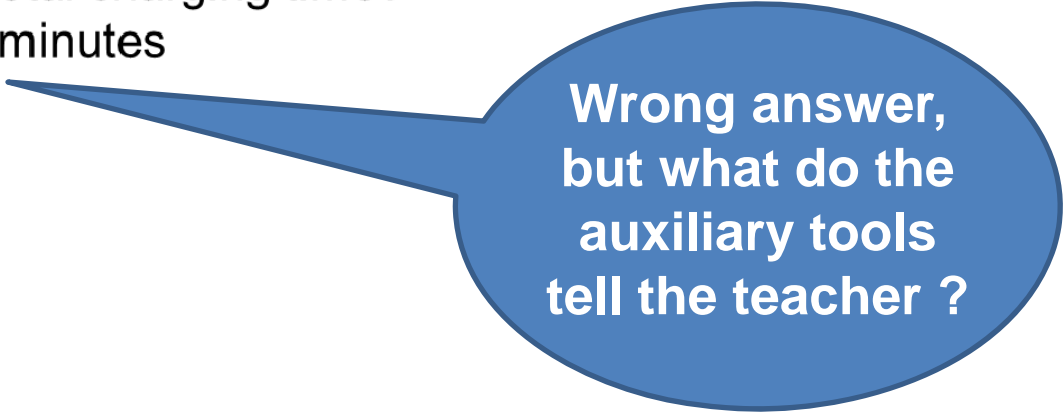
Example 2

Problem 5

In 24 minutes the battery is charged for 75%.

What is the total charging time?

Answer: 30 minutes



Wrong answer,
but what do the
auxiliary tools
tell the teacher ?

Example 2

Problem 5

In 24 minutes the battery is charged for 75%.
 What is the total charging time?

Answer: 30 minutes



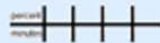
scrap paper empty



scrap paper with grid



bar



table

75	50	25
24	12	6

Example 3

***Advanced mathematical
thinking by low attaining
students***



Pergamon

Journal of Mathematical Behavior
20 (2002) 461–475

**Mathematical
Behavior**

Instances of mathematical thinking among low attaining students in an ordinary secondary classroom

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Abstract

This paper is a report of a classroom research project whose aim was to find out whether low attaining 14-year-old students of mathematics would be able to think mathematically at a level higher than recall and reproduction during their ordinary classroom mathematics activities. Analysis of classroom interactive episodes revealed many instances of mathematical thinking of a kind which was not normally exploited, required or expected in their classes. Five episodes are described, comparing the students' thinking to that usually described as "advanced." In particular, some episodes suggest the power of a type of prompt which can be generalized as "going across the grain." © 2002 Elsevier Science Inc. All rights reserved.

Example 3

“Going across the grain” study by Watson (2002)

“ ‘Low attaining students’ are generally classified [...] on the basis of accumulated incompetence in tests and other written work.”

~~***Deficiency-based approach***~~

Proficiency-based approach

Watson

Example 3

$5 \times 7 =$	$7 \times 5 =$	$35/5 =$	$35/7 =$
$6 \times 7 =$	$7 \times 6 =$	$42/6 =$	$42/7 =$
$7 \times 7 =$	—	$49/7 =$	—
$8 \times 7 =$	$7 \times 8 =$	$56/8 =$	$56/7 =$

$$23 \times 7 = 161$$

“All could do this after some thought, although their previous patterns working down the page did not help them in this case.”

- identify and use patterns
- work with abstractions and relations

Research on MLD needs a
proficiency-based approach

It is time to reveal
what students with MLD know,
rather than what they do not know

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